

Bilkent University Department of Mathematics

Senior Projects

Math 491/492

2025-Spring

Course Committee: Ö. Ünlü, G. Yıldırım, and D. Yılmaz

Course Coordinator: G. Yıldırım

Senior Project Presentations

Tuesday, May 20, in the seminar room (SA-141)13:00–13:30: Muhammet Emin Ovat - (Yılmaz)The Complex Character Ring

13:30–14:00: Kazım Tuğşad Koyuncu -(Yılmaz) - Units of Burnside Rings

Break 14:00-14:15

14:15–14:45: Ayşe Verda Irmak- (Maronikolakis)Hypercyclicity of Adjoint Multipliers in Hardy Spaces

14:45–15:15: Behzar Deniz Özyörük - (Okay) - Bundle scenarios for contextuality

Seminar Talks

Topology Seminar-May 12: Adem Eren Uyanık - (Berktav)- Introduction to Bicategories and Extended Topological Field Theories

The Complex Character Ring (math 491)

Muhammet Emin Ovat

Supervisor: Deniz Yılmaz

The theory of biset functors, introduced by Serge Bouc, provides a powerful framework in representation theory. In this presentation, we explore the character ring of a finite group as a biset functor. The exposition is divided into three parts. First, we introduce fundamental concepts of representation theory—group representations, characters, and $\mathbb{C}G$ -modules. Next, we develop the biset functor framework, introducing induction, restriction, inflation, deflation, and conjugation operations, and give a parametrization of simple biset functors. Finally, we present Bouc's work on the complex character ring functor and prove that it decomposes as a direct sum of simple biset functors parametrized by the pairs of a cyclic group G and a primitive character of the automorphism group of G.

Units of Burnside Rings

Kazım Tuğşad Koyuncu

Supervisor: Deniz Yılmaz

Let G be a finite group. The Burnside ring B(G) of G is a fundamental object in representation theory. It is a commutative ring where the multiplication is induced from the direct products of G-sets and addition is from the disjoint union of G-sets. In this project our first aim is to study Dress' result characterizing solvable groups in terms of the Burnside rings. Dress proved in 1969 that G is solvable if and only if the prime spectrum of B(G) is connected in Zariski topology. After studying the proof of this result, we will concentrate the units of the Burnside ring. Another important result of Dress is that the unit group of the Burnside ring of a group of odd order has cardinality 2. Together with the result above, this gives an equivalent statement of odd order theorem in terms of the units of the Burnside rings: A group of odd order is solvable if and only if the cardinality of the units of the Burnside ring is equal to 2.

Hypercyclicity of Adjoint Multipliers in Hardy Spaces

Ayşe Verda Irmak

Supervisor: Konstantinos Maronikolakis

We study adjoint multiplication operators on the Hardy space H^2 and identify when these operators are *hypercyclic*, meaning that a vector's orbit under the operator is dense in the space. Using the Birkhoff Transitivity Theorem and the Godefroy–Shapiro Criterion, two classical tools in linear dynamics, we first analyze constant multiples λB and polynomial multiples p(B) of the backward shift on ℓ^2 . We then carry these ideas to the Hardy Space H^2 by linking sequences with holomorphic functions. For a bounded holomorphic function φ on the unit disc, the multiplier M_{φ} acts by $M_{\varphi}f = \varphi f$. Point evaluations show that no orbit can be dense, so M_{φ} is never hypercyclic. To look for richer dynamics we pass to the Hilbert-space adjoint M_{φ}^* , called an *adjoint multiplier*. The main result that we study states that for any non-constant bounded holomorphic function φ on the unit disc, the adjoint multiplier M_{φ}^* is hypercyclic if and only if $\varphi(\mathbb{D})$ intersects the unit circle.

Bundle scenarios for contextuality

Behzat Deniz Özyörük

Supervisor: Cihan Okay

Probability distributions obtained from experiments in quantum mechanics do not always admit a joint probability distribution. These distributions are called contextual distributions. In [1], authors introduced the category Scen and gave a sheaf theoretic reformulation of contextuality. Later, authors in [2] embedded this category and the notion of contextuality into categories bScen and sScen. These categories use the theory of simplicial complexes and simplicial sets. In this project, we start by introducing a more primitive category called Scent which is based on sets. We study its properties along with contextuality. We conclude that this category does not admit contextuality and therefore we move on to study bScen and sScen.

References

[1] R. S. Barbosa, M. Karvonen, and S. Mansfield, "Closing Bell: Boxing black box simulations in the resource theory of contextuality," in Samson Abramsky on Logic and Structure in Computer Science and Beyond (A. Palmigiano and M. Sadrzadeh, eds.), vol. 25 of Outstanding Contributions to Logic, Springer, 2023.

[2] Rui Soares Barbosa, Aziz Kharoof, and Cihan Okay. A bundle perspective on contextuality: Empirical models and simplicial distributions on bundle scenarios. arXiv preprint arXiv:2308.06336, 2023. doi: 10.48550/arXiv.2308.06336.

Introduction to Bicategories and Extended Topological Field Theories

Adem Eren Uyanık

Supervisor: Kadri İlker Berktav

In this talk, I will motivate and introduce *bicategories* and *ex*tended Topological Field Theories (eTFTs). I will start with recalling the basics of Topological Quantum Field Theories (TQFTs) and the classification theorem in 2-dimensional case. I will then give the definition of eTFT as a functor between particular bicategories, generalizing the case of ordinary TQFT. Lastly, I will informally discuss one of the classification theorems for 2-dimensional eTFTs.